

## UNIT-VII

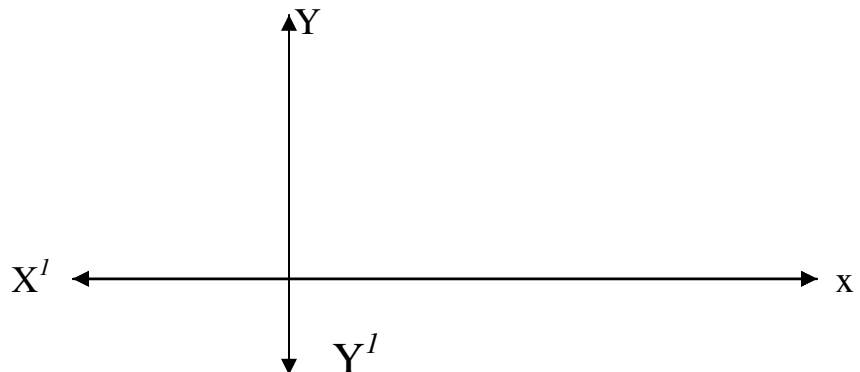
### (Application of Derivatives)

#### Group – A

Answer the following questions.

1 X 20 = 20

1. Find the min. value of  $|x+5|$  where  $x \in \mathbb{R}$
2. Write the equation of the tangent for the function  $y = f(x)$  at the point  $(x_1, y_1)$ .
3. For a curve  $y = f(x)$ , if  $f'(x) = 3x - 2$  then find the slope of the tangent. When  $x = -1$ .
4. What should be the value of  $f'(a)$  for  $f(x)$  to have maximum or minimum value at  $x = a$ .
5. Find the minimum value if any of the function  $f(x) = x^2$  where  $x \in \mathbb{R}$ .
6. Find the slope of the tangent to the curve  $y = x^3 - x$  at  $x = 2$ .
7. A line is parallel to x-axis. What is the slope of the line ?
8. If two lines (with slope  $m_1$  &  $m_2$ ) are perpendicular to each other then write the relation between their slopes.
9. Find the maximum value of the function  $f(x) = |\sin x + 5|$
10. If  $A$  is the area and  $r$  is the radius of the circle then find the rate of change of area with respect to  $r$ .
11. Define monotonic function ?
12. Write the equation of the normal for the function  $y = f(x)$  at the point  $(x_1, y_1)$ .
13. Define increasing function ? if  $f(x) = 3x, \forall x \in \mathbb{N}$  then  $f(x)$  is increasing function or not ?
14. Prove that  $f(x) = \sin x$  is increasing in  $[0, \pi/2]$
15. Define decreasing function ?
16. What is the slope of the line  $y = -3x + \frac{1}{2}$
17. State whether the given function of the graph is increasing or decreasing.



18. If  $x_1 < x_2 \implies f(x_1) < f(x_2)$  for all  $x_1, x_2 \in \mathbb{R}$  then that function is called ?
19. If two lines (with slope  $m_1$  &  $m_2$ ) parallel to each other then what is the relation between their slopes.

20. If the slope of the tangent of the function  $y = f(x)$  at any point  $(x_1, y_1)$  is  $-2$  then what will be the slope of the normal at the same point.

**GROUP – B**

**15 X 4 = 60**

1. For the given function  $f(x) = x^2 + 2x - 8, x \in [-4, 2]$   
Using property
  - (a) Check continuity at  $[-4, 2]$ .
  - (b) Check Differentiability at  $[-4, 2]$ .
  - (c) Show  $-f(-4) = f(2)$
  - (d) State the above function  $f(x)$  satisfy or does not Satisfy Rolle's theorem
  
2. For the given function  $y = x^2 - 2x + 7$ 
  - (a) Find  $dy/dx$
  - (b) Find  $[dy/dx]_{(2, 1)}$
  - (c) Find equation of tangent at point  $(2, 1)$
  - (d) Find equation of normal at point  $(2, 1)$
  
3. The volume of a cube is increasing at the rate of 9 cubic centimeter/sec. How fast is the surface area increasing when the length of edge is 10 c.m.
  
4. The volume of spherical balloons is increasing at the rate of  $27 \text{ cm}^3/\text{sec}$ . Find the rate of change of its surface area when its radius is 8 cm.
  
5. Verify Lagrange's mean value theorem for  $f(x) = x^2 + x - 1$  on  $[0, 4]$
  
6. Find the interval in which the function  $f(x) = x^2 - 4x + b$  is –
  - (a) Strictly increasing
  - (b) Strictly decreasing
  
7. Show that the function  $f(x) = x - 1/x$  is increasing for  $x \in \mathbb{R}$  (1+1½+1½)
  
8. Prove that the curves  $x = y^2, xy = K$  Cut at right angle if  $8K^2 = 1$ .  
(1+1+2)
  
9. Find the approximate value of  $(25)^{1/3}$  Using differentiation. (1+1+2)
  
10. Find the local maxima on local minima of the function  $f(x) = x^3 - 6x^2 + 9x + 15$   
(1+1+2)
  
11. If  $f(x) = 3x^2 + 15x + 5$  find the approximate value of  $f(3.02)$  Using differentiation. (1+1+1+1)
  
12. Find two number whose sum is 24 and whose product is as large as possible.  
(1+1+1+1)
  
13. Find the maximum profit that a company can make if the profit function is given by  $p(x) = 41 - 72x - 18x^2$  (1+1+1+1)

14. Verify Rolle's theorem for the function  $f(x) = \sin x + \cos x$  in  $[0, \pi/2]$  (1+1+1+1)
15. Find the equation of the tangent and normal to the curve  $y = x^2 + 4x + 1$  at the point  $x = 3$  (1+1+1+1)

**Group - C**

**6 X 10 = 60**

1. For a given function  $y = f(x) = \sin x + \cos x, 0 < x < \pi/2$
- i. Find  $dy/dx$  1
  - ii. Find the point at which  $dy/dx = 0$  1
  - iii. Find  $d^2y/dx^2$
  - iv. Find  $d^2y/dx^2$  when  $dy/dx = 0$  1
  - v. Find the point of maxima if exist 1
  - vi. Find the point of minima if exist 1
2. If  $y = x^x$  then find
- i.  $dy/dx$  1
  - ii. The point at which  $dy/dx = 0$  1
  - iii. Find  $d^2y/dx^2$  1
  - iv. Evaluate  $d^2y/dx^2$  at the point where  $dy/dx = 0$  1
  - v. Find the point of local maxima or local minima if exist 1
  - vi. Find the value of local maxima or local minima if exist 1
3. Find the interval on which the function  $y = f(x) = (x + 1)^3 \cdot (x - 3)^3$  is  
(a) increasing (b) decreasing
4. Find the coordinates of the point on the curve  $y = x^2 + 3x + 4$ , the tangent at which passes through origin.
5. Show that all the rectangles inscribed in a given fixed circle the square has the maximum area.
6. Prove that the volume of the largest cone that can be inscribed in a sphere of a radius  $r$  is  $8/27$  of the volume of the Sphere.
7. Find the equations of the tangent and normal to the curve  $x = \theta + \sin \theta$ ,  $y = 1 + \cos \theta$  at  $\theta = \pi/4$
8. Prove that the curve  $(x/a)^n + (y/b)^n = 2$  touches the straight line  $x/a + y/b = 2$  at the point  $(a, b)$  what even the value of  $n$ .
9. Find the intervals in which function  $f(x) = \sin x + \cos x, 0 \leq x \leq 2\pi$  is strictly increasing or strictly decreasing.
10. If the length of the simple pendulum is decreased by 2% find the percent decreased in its period  $T$  where  $T$  is equal to :-

$$T = 2\pi \sqrt{l/g}$$