UNIT-V

(TOPIC – DETERMINANT)

1 MARKS QUESTION

1. If $\begin{vmatrix} x & 4 \\ 9 & x \end{vmatrix} = 0$. Find x. 1

2. Find the value of $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \\ 9 & 12 & 15 \end{vmatrix}$. 1

3. Evaluate $\begin{vmatrix} 1 & -i \\ i & 1 \end{vmatrix}$ 1

4. Let A be a 3 X 3 matrix such that $|A|$ = 5 then find the value of $|2A|$. 1

5. Evaluate $\begin{vmatrix} 1 & 1 & 2 \\ x & y & x+y \\ x & x^2 & x+x^2 \end{vmatrix}$. 1

6. Evaluate $\begin{vmatrix} \cos 15^0 & \sin 45^0 \\ \sin 15^0 & \cos 45^0 \end{vmatrix}$ 1

7. Use cofactors of elements of 2nd row, evaluate $\begin{vmatrix} 1 & 2 & 3 \\ 0 & 0 & 4 \\ 1 & 2 & 1 \end{vmatrix}$. 1

8. Prove that $\begin{vmatrix} x^2+xy+y^2 & x+y \\ x^2-xy+y^2 & x-y \end{vmatrix}$ = $-2y^3$. 1

9. Prove that $\begin{vmatrix} 1 & \log_b^a \\ \log_a^b & 1 \end{vmatrix}$ = 0 1

10. For what value of x the matrix $\begin{pmatrix} 1 & 2 \\ x & 2 \end{pmatrix}$ is singular. 1

11. In the matrix $\begin{pmatrix} 1 & 3 & 5 \\ 4 & -5 & 2 \\ 3 & 2 & 6 \end{pmatrix}$ find C_{23}. 1

12. Prove that $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-b & a-b & b-c \end{vmatrix}$ = 0 1

13. If A = $\begin{pmatrix} 0 & i \\ i & 1 \end{pmatrix}$ and B = $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Find the value of $|A|+|B|$. 1

14. Evaluate $\begin{vmatrix} 0 & 3 & 2 \\ 0 & 4 & 6 \\ 7 & 2 & 9 \end{vmatrix}$. 1

15. Evaluate $\begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{vmatrix}$. 1

16. If A is 3 X 3 matrix such that $|A| = \lambda$ then find the value of $|-A|$. 1

17. Write the value of determinant $\begin{vmatrix} 2 & 5 & 7 \\ 1 & 2 & 3 \\ 6x & 15x & 21x \end{vmatrix}$. 1

18. Let A = $(dij)_{3X3}$ and Cij cofactor of aij. Write the value of $a_{11}c_{21}+a_{12}c_{22}+a_{13}c_{23}$. 1

19. A is skew symmetric matrix of order 3, write the value of $|A|$ 1

20. If A is a square matrix such that A(adjA) = $\begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix}$ then write the value of $|adjA|$. 1

4 MARKS QUESTIONS

1. Using determinant, Find the area of triangle whose vertices are (2, 7), (1, 1) and (10, 8). 2+2
   If $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are vertices of Δ (Triangle). 2+2
2. i) Write formula to find area of triangle (Δ) in determinant form.
   ii) What do you conclude if Δ = 0
3. Find the value/values of K if the area of triangle with vertices (-2, 0), (0, 4) and (0, K) is 4 square unit. 4
4. If the points (a, b), (a', b') and (a –a', b –b') are collinear, use determinant to prove that ab' = a'b. 4
   If A is a matrix of order 3X3 and $|A|$ = 4 then Find 4
   i) $|adjA|$
5. ii) $|A.adjA|$
   iii) $|2A|$
   iv) $|adj(adjA)|$
   Prove by using properties of determinant. 4
6. $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ = (a – b) (b – c) (c – a).
   4
7. Find value of x if $\begin{vmatrix} x-3 & x & x \\ x & x-3 & x \\ x & x & x-3 \end{vmatrix}$ = 0.
   Prove without expanding at any stage 4
8. $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2. \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$
   4
9. Prove that $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3$

10. Using properties of determinant prove that $\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$ = 4abc 4

11. Prove that by using properties $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$ = (a – b) (b – c) (c – a) (a + b + c) 4

12. Prove that $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & 2x+4 \end{vmatrix}$ = $(5x + 4)(4 - x)^2$. 4

13. Show that the value of the determinant $\begin{vmatrix} 0 & \tan x & 1 \\ 1 & -\sec x & 0 \\ \sec x & 0 & \tan x \end{vmatrix}$ is independent of x. 4

14. Prove that $\begin{vmatrix} y+z & x+y & x \\ z+x & y+z & y \\ x+y & z+x & z \end{vmatrix}$ = $x^3 + y^3 + z^3 - 3xyz$ 4

15. If a, b, c are A.P. then find the value of $\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$ 4

16. Find A^{-1} if exist, where A = $\begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ 4

17. Show that the following system of homogeneous equation have non-trivial solution. 4
2x + y + z = 0
3x + 2y + 2z = 0
X + 2y + 2z = 0. Hence find its solution.

18. Using matrix solve the system of linear equation 4
3x + 6y = 5
3x + 4y = 10

19. Find the adjoint of matrix $\begin{pmatrix} 2 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 2 & 1 \end{pmatrix}$ 4

20. For matrix A = $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$. Show that A.(adjA) = $|A|.I$ 4

<u>6 MARKS QUESTIONS</u>

1. Show that $\begin{vmatrix} \sin^2 A & \sin A & \cos^2 A \\ \sin^2 B & \sin B & \cos^2 B \\ \sin^2 C & \sin C & \cos^2 C \end{vmatrix}$ = $-(\sin A - \sin B).(\sin B - \sin C).(\sin C - \sin A)$ 6

2. Prove that $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$ 6

3. Using properties of determinants, show that $\begin{vmatrix} a^2+1 & ab & ac \\ ba & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = a^2+b^2+c^2+1$ 6

4. Evaluate the Determinant 6

$\Delta = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$. Also prove that $2 \le \Delta \le 4$.

5. Solve the following system of linear equation by using matrices. 6

$x+2y+3z=14$

$x+y+z=6$

$x-2y-z=-6$

6. Show that the following system of linear equations is consistent 6

$2x-y+3z=5$

$3x+2y-z=7$

$4x+5y-5z=9$

Find the solution.

7. If A = $\begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$ show that $A^{T}.A^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$ 6

8. If $x \ne y \ne z$ then show that $1+xyz=0$ 6

where, $\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$

9. If $A = \begin{pmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{pmatrix}$, find AB, without using A^{-1}, solve the system of equations : 6

$x-y=3$

$2x+3y+4z=17$

$y+2z=7$

10. Prove that $\begin{vmatrix} x2 & y2 & z2 \\ yz & zx & xy \\ x & y & z \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$ 6

11. The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method. 6

12. A manufacturer produce three products x, y, z which he sells in two markets. Annual sales are indicated below : 6

		Product	
Market	x	y	z
I	5,000	2,000	8,000
II	6,000	3,000	6,000

a) If unit sale prices of x, y, z are ₹ 3.0, ₹ 2.0 and ₹ 2.5 respectively. Find the total revenue in each market with the help of matrix algebra.

b) If the unit cost of above three commodities are ₹ 2.00, ₹ 1.5 and ₹ 1.0 respectively. Find the gross profit.