UNIT-XIII

LINER PROGRAMMING

General Instruction :-

(i) All question are compulsory.

(ii) The question paper consists of 45 question divided into three section A, B and C. Section A comprise of 20 question of one mark each, section B comprises of 15 question of four marks each and section C comprises of 10 question of six marks each.

(iii) All question is section A are to be answer in one word, one sentence of a per the exact requirement of the question.

Marking scheme :-

(i) Marking scheme for section B :-

(a) Framing equation from the given constraints – 1 Mark

(b) Sketching these equation on the graph - 1 Mark

(c) Finding the feasible region and the corner points – 1 Mark

(d) Finding values of Z at these points and see at which point Z is minimum/ maximum - 1 Mark

4 Marks

(ii) Marking scheme for section C :-

(a) Mathematical formulation of the given problem – 2 Mark

(b) Sketching these equation on graph - 2 Mark

(c) Finding the feasible region and the corner points – 1 Mark

(d) Finding values of Z at these points and see at which point Z is minimum/ maximum - 1 Mark

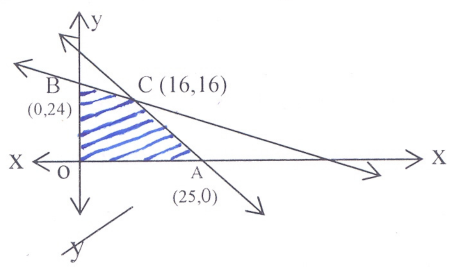
6 Marks

Section – A

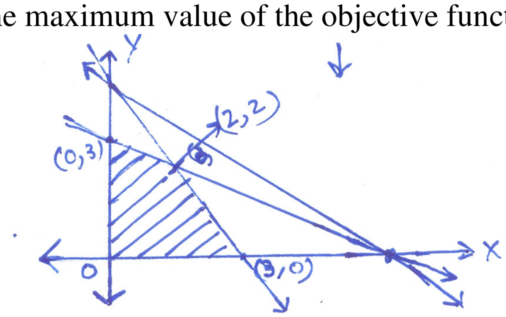
1. State whether the region represented by $x \geq o, y \geq o.\ x+y \leq 1$ bounded or unbounded. 1
2. What are non-negativity constraints in an LPP. 1
3. Draw a rough sketch of the region represented by. 1

   $x \geq o, y \geq o,\ x \leq 2,\ y \leq 3$ o

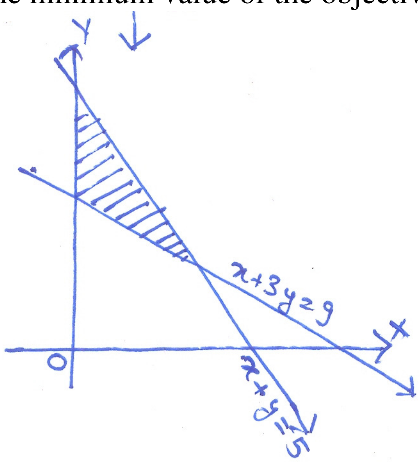
4. Draw the graph of the inequation $2x+3y \geq 6$, $x \geq o$, $y \geq o$. 1
5. The optimal solution of an LPP occurs at two distinct points (x, y) and (x_2, y_2) of the feasible region. Will this solution occur at some other points also? 1
6. Determine the maximum value of $Z= 4x+3y$ if the feasible region for an LPP is the shaded region as shown in the figure. 1



7. For an LPP, the feasible region is shown shaded in the figure below. Find the maximum value of the objective function $Z = 5x+7y$. 1



8. The feasible region for an LPP is shown shaded in the figure below. Find the minimum value of the objective function $Z = 11x+7y$. 1



9. The corner points of the feasible region for an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). Let $f = 4x+6y$ be the objective function then find maximum f and minimum f. 1

10. Define linear programming problem (LPP)? 1

11. What do you mean by an objective function of an LPP? 1

12. What do you mean by the feasible solution of an LPP? 1

13. What are decision variable in an LPP? 1

14. What do you mean by the optional solution of an LPP? 1

15. The corner points of the feasible region determined by the system of linear constraints ate (0,10) , (5, 5), (15, 15) and (0, 20). Let $Z = px+qy$. where $p,q > 0$. Find condition on point q so that the maximum of Z occurs at both the points (15, 15) and (0, 20). 1

16. Region represent by $x \geq 0, y \geq 0$ is ……………………….. . 1

17. Region represent by $x \leq 0, y \leq 0$ is ……………………….. . 1

18 The maximum or minimum value of an objective function of an LPP is ……………................ solution. 1

19. In an LPP, the objective function is always …………………….. 1

Section – B

1. Solve the following LPP, 1+1+1+1

   Maximise $Z = 3x+4y$,

   Subject to $x+y \leq 4$, $x \geq 0$, $y \geq 0$.

2. Solve the following LPP, 1+1+1+1

   Manimise $Z = 3x+4y$,

   Subject to $3x+2y \leq 12$, $x + 2y \leq 8$, $x \geq 0$, $y \geq 0$.

3. Solve the following LPP, 1+1+1+1

   Maximise $Z = 5x+3y$,

   Subject to $3x+5y \leq 15$, $5x + 2y \leq 10$, $x \geq 0$, $y \geq 0$.

4. Solve the following LPP, 1+1+1+1

   Maximise $Z = 3x+5y$,

   Such that $x+3y \geq 3$, $x + y \geq 2$, $x, y \geq 0$.

5. Solve the following LPP, 1+1+1+1

   Maximise $Z = 3x+2y$,

   Subject to $x+2y \leq 10$, $3x + y \leq 15$, $x , y \geq 0$.

6. Solve the following LPP, 1+1+1+1

   Maximise $Z = x+2y$,

   Subject to $2x+y \geq 3$, $x + 2y \geq 6$, $x , y \geq 0$.

7. Solve the following LPP, 1+1+1+1

   Minimise and Maximise $Z = 5x+10y$,

   Subject to $x+2y \leq 120$, $x + y \geq 60$, $x - 2y \geq 0$, $x \geq 0$, $y \geq 0$.

8. Solve the following LPP, 1+1+1+1

Minimise and Maximise $Z = x+2y$,

Subject to $x+2y \geq 100$, $2x - y \leq 0$, $2x + y \leq 200$; $x, y \geq 0$.

9. Solve the following LPP, 1+1+1+1

Maximise $Z = -x+2y$,

Subject to $x \geq 3$, $x+y \geq 5$, $x + 2y \geq 6$, $y \geq 0$.

10. Solve the following LPP, 1+1+1+1

Maximise $Z = x+y$,

Subject to $x-y \leq -1$, $-x + y \leq 0$, $x , y \geq 0$.

11. Solve the following LPP graphically; 1+1+1+1

Maximise $Z = 4x+y$,

Subject to $x+y \leq 50$, $3x + y \leq 90$; $x,y \geq 0$.

12. Solve the following LPP graphically; 1+1+1+1

Minimise $Z = 200x+500y$,

Subject to $x+2y \geq 10$, $3x +4y \leq 24$; $x,y \geq 0$.

13. Solve the following LPP graphically; 1+1+1+1

Minimise and Maximise $Z = 3x+9y$,

Subject to $x+3y \leq 60$, $x + y \geq 10$, $x \leq y$; $x,y \geq 0$.

14. Solve the following LPP graphically; 1+1+1+1

Maximise $Z = -50x+20y$,

Subject to $2x-y \geq -5$, $3x + y \geq 3$, $2x - 3y \leq 12$; $x, y \geq 0$.

15. Solve the following LPP graphically; 1+1+1+1

Minimise $Z = 3x+2y$,

Subject to $x + y \geq 8$, $3x + 5y \leq 15$; $x , y \geq 0$.

Group – C

1. A furniture dealer deals in only two items; table and chairs. He has ₹ 5000 to invest and a space to store at most 60 pieces. A table cast him ₹ 250 and a chair ₹ 50. He can sell a table at a profit of ₹ 50 and a chair at a profit of ₹ 15. Assuring that he can sell all the items that he buys, how should he invest his money in order that the may maximize his profit? 2+2+1+1

2. If a young man rides his motor cycle at 25 km/hr, he has to spend ₹ 2 per kilometer on petrol; if he rides it at a faster speed of 40 km/hr, the petrol costs increases to ₹ 5 per kilometer. He has ₹ 100 to spend on petrol and wishes to find the maximum distance he can travel within one hour. Express this as a LPP and then solve it. 2+2+1+1

3. Suppose every gram of wheat provides 0.1gm of proteins and 0.25gm of carbohydrates and the corresponding values for rice are 0.05gm and 0.5gm respectively. Wheat costs ₹ 5 and Rice ₹ 20 per kilogram. The minimum 2+2+1+1

daily requirements of proteins and carbohydrates for an average man are 50gm and 200gm respectively. In what quantities should wheat and rice be mixed in the daily diet to provide the minimum daily requirements of proteins and carbohydrates at minimum cost, assuming that both wheat and rice are to be taken in the diet?

4. A chemical industry produces two compounds, A and B. The following table gives the units of ingredients C and D (per kg) of compounds A and B as well as minimum requirements of C and D and costs per kg of A and B. 2+2+1+1

	Compound (in units)		**Minimum requirements**
	A	**B**	
Ingredient C (per kg)	1	2	80
Ingredient D (per kg)	3	1	75
Cost per kg (in Rs.)	4	6	

Find the quantities A and B which would minimize the cost.

5. One kind of cake requires 200gm of flour and 25gm of fat and another kind of cake requires 100 gm of flour and 50gm of fat. Find the maximum number of cakes which can be made from 5gm of flour and 1gm of fat, assuming that there is no shortage of the other ingredients used in making the cakes. 2+2+1+1

6. A company has factories located at each of the two places P and Q. From these location, a certain commodity is delivered to each of the three depots situated at A, B and C. The weekly requirements of the depots are respectively 7, 6 and 4 units of the commodity while the weekly production capacities of the factories at P and Q are respectively 9 and 8 units. The cost of transportation per unit is given below. 2+2+1+1

To	**Cost (in Rs.)**		
From	**A**	**B**	**C**
P	16	10	15
Q	10	12	10

How many units should be transported from each factory to each depot in order that the transportation cost is minimum. Formulate the LPP mathematically and then solve it.

7. An aeroplane can carry a maximum of 200 passengers. A profit of ₹ 1000 is made on each executive class ticket and a profit of ₹ 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least four times as many passengers prefer to travel by economy class than by executive class. Determine how many tickets of each type must be sold in order to maximize the profit for the airline. What is the maximum profit? 2+2+1+1

8. A dietician wishes to mix together two kinds of food X and Y such a way that the mixtures contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin contents of 1 kg food is given below. 2+2+1+1

Food	**Vitamin A**	**Vitamin B**	**Vitamin C**
X	1	2	3
Y	3	2	1

One kg of food X costs Rs. 16 and one kg of food Y costs Rs. 20. Find the least cost of the mixture which will produce the required diet.

9. A firm manufactures two types of products A and B and sells them at a profit of Rs. 5 per unit of type A and Rs. 3 per unit of type B. Each products is processed on two machines M_1 and M_2. One unit of type A requires one minute of processing time on M_1 and two minutes of processing time on M_2 whereas one unit of type B requires one minute of processing time on M_1 and one minute on M_2. Machines M_1 and M_2 are respectively available for almost 5 hours and 6 hours in a day. Find out how many units of each type of product should the firm produce a day in order to maximize the profit. Solve the problem graphically. 2+2+1+1

10 A dealer wishes to purchase a number of fans and sewing machines. He has only Rs. 5760 to invest and has space for at most 20 items. A fan costs him Rs. 360 and a sewing machine Rs. 240. He expects to gain Rs. 22 on a fan and Rs. 18 on a sewing machines. Assuming that he can sell all the items he can buy, how should he invest the money in order to maximize the profit? 2+2+1+1