

## UNIT-XII

### Mathematics (Three Dimensional Geometry)

**This group Contains 20 questions carrying 1 (one) mark each.**

- 1 Find the distance between two planes  $2x + 3y + 4z = 4$  and  $2x + 3y + 4z = 12$  1
- 2 If a line makes angles  $90^\circ$ ,  $135^\circ$  and  $45^\circ$  with the x,y and z- axes respectively, find its direction Cosines. 1
- 3 If a line has the d.r.'s 1,2,4 then what are its direction Cosines ? 1
- 4 Write down the condition for perpendicularity of two straight lines having d.r.'s  $(a_1, b_1, c_1)$  and  $(a_2, b_2, c_2)$ . 1
- 5 Write down the condition for parallelism of two straight lines having d.r.'s  $(a_1, b_1, c_1)$  and  $(a_2, b_2, c_2)$ . 1
- 6 Write down the formula for shortest distance between two skew lines  $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$  and  $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ . 1
- 7 Write down the shortest distance between two parallel lines  $\vec{r} = \vec{a}_1 + \lambda \vec{b}$  and  $\vec{r} = \vec{a}_2 + \mu \vec{b}$ . 1
- 8 Find the d.r.'s of  $\frac{x-2}{2} = \frac{y-1}{5} = \frac{2z-6}{4}$ . 1
- 9 Write down the equation of the plane  $6x+4y+3z=12$  in intercept form. 1
- 10 Write down the formula of distance from a point  $(z_1, y_1, z_1)$  of the plane  $Ax+By+Cz + D = 0$  1
- 11 Define skew lines. 1
- 12 Write down the d.c.'s of Co-ordinate axes. 1
- 13 Write down the formula for angle between a line  $\vec{r} = \vec{a} + \lambda \vec{b}$  and a plane  $\vec{r} \cdot \vec{n} = d$ . 1
- 14 Determine the d.R.s of the normal to the plane  $x+y+z=1$ . 1
- 15 Write down the intercept form of the equation of a plane. 1
- 16 Write down the Cartesian equation of the plane. 1
- 17 Write the equation of a line passing through two given points  $A(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$  1
- 18 Write down the d.R.s of a line passing through two pts.  $A(x_1, y_1, z_1)$  and  $B(x_2, y_2, z_2)$  1
- 19 If a line has d.r.'s 2,-1,-2, write the d.r.'s. of a line  $l$  to this line. 1
- 20 Write down the equation of plane  $\vec{r} \cdot (\vec{i} + 2\vec{j} - \vec{k}) = 3$  into cartesian form. 1

**This group Contains 15 questions carrying 4 mark each.**

- 1 Find the direction cosine of a line which makes equal angles with co-ordinate axes. 4

- 2 If Direction ratios of three lines are 12,-3,-4 ; 4,12,3 ; & 3,-4,12 respectively then
- Find Direction cosine of all lines 4
  - Prove that 1<sup>st</sup> and 2<sup>nd</sup> lines are  $\perp$  4
  - Prove that 2<sup>nd</sup> and 3<sup>rd</sup> lines are not  $\perp$  4
  - Find angle between 1<sup>st</sup> and 3<sup>rd</sup> lines
- 3 If A(1,-1,2) , B (3,4,-2) , C (0,3,2) & D (3,5,6) are any four points then.
- Find D.R. of AB
  - Find D.R. of CD 4
  - Find angle between AB and CD.
  - Find D.C. of AB
- 4 Show that line through the points A (4,7,8), B (2,3,4) is parallel to the line through the pts C (-1,-2,1), D (1,2,5) 4
- 5 If a line makes  $\alpha, \beta, \gamma$  with X-axis, Y-axis and Z-axis respectively, then find the value of  $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ . 4
- 6 If Equation of a line AB is  $\frac{x+3}{3} = \frac{y-4}{5} = \frac{y+8}{6}$  & C (-2,4,-5) is any point, Then
- Write down the direction ratio of AB 4
  - Write down the direction cosine of AB
  - Write the cartesian equation of a line through C &  $\perp$  to AB
  - Write vector equation of AB
- 7 If  $\vec{r} = 2i' + (3-2\lambda)j' + (7\lambda - 5)k'$  is equation of a line then.
- Write down the direction ratio of the line
  - Write down the Co-ordinate of a pt. which lie on the line 4
  - Cartesian equation of line = ?
  - Direction cosine of line
- 8 Find the value of p so that the lines  $\frac{1-x}{3} = \frac{y-2}{2p} = \frac{z-3}{2}$  and  $\frac{7-7x}{7} = \frac{y-5}{1} = \frac{2z-6}{2p}$  are  $\perp$  to each other. 4
- 9 Find the angle between the lines  $\vec{r} = 2i' - 5j' + k' + \lambda(3i' + 2j' + 6k')$  and  $\vec{r} = 7i' - 6k' + \mu(i' + 2j' + 2k')$  4
- 10 If  $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$  is any line and  $10x + 2y - 11z = 3$  is any plane, then
- Find Direction ratio of the line
  - Direction ratio of Normal to the plane
  - If  $\theta$  is angle between the line & plane then write down the formula for  $\sin\theta$  4
  - Find the angle between the given line & plane

- 11 If a plane cuts X-axis at A, Y-axis at B & Z-axis at C having intercepts 2,3, & 4 on X,Y & Z axis respectively ; Then
- Find equation of plane in intercept form 4
  - Find Co-Ordinate of centroid of  $\Delta ABC$
  - Find D.R. of AB
  - Find D.C. of AB
- 12 Find the equation of the plane through the intersection of the planes  $3x-y+2z-4=0$  and  $x+y+z+7=0$  and through the point  $(2,2,1)$ . 4
- 13 Find the equation of the plane, which contains the line of intersection of the plane  $\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0$ ,  $\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5 = 0$ , and which is tr to the plane  $\vec{r} \cdot (5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0$ . 4
- 14 If a line makes an angle  $\frac{\pi}{4}$  with  $\hat{i}$ ,  $\frac{\pi}{3}$  with  $\hat{j}$ , then Find the acute angle between line &  $\hat{k}$ . 4
- 15 If  $(a_1, b_1, c_1)$  – Direction ratio of 1<sup>st</sup> line &  $(a_2, b_2, c_2)$  – Direction ratio of 2<sup>nd</sup> line and  $\theta$  is angle between them, then
- Write down the formula for  $\sin\theta$  4
  - Write the condn for both lines are  $\parallel$
  - Write down the condn for perpendicularity
  - Write down the formula for  $\cos\theta$

**Each of the following questions contain 6 marks**

- 1 Prove that if a plane has the intercepts a,b,c and is at a distance of p units from the origin then :- 6
- $$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2}$$
- 2 Find the vector equation of the line passing through the point  $(1,2,-4)$  and  $\perp$  to the two lines :- 6
- $$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \quad \text{and} \quad \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$
- 3 Find the vector equation of the line passing through  $(1,2,3)$  and parallel to the planes  $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$  and  $\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$ . 6
- 4 Find the equation of the plane passing through the point  $(-1,3,2)$  and  $\perp$  to each of the planes  $x+2y+3z=5$  and  $3x+3y+z=0$  6
- 5 Find the Co-ordinates of the point where the line through  $(5,1,6)$  and  $(3,4,1)$  crosses the zx plane. 6
- 6 Find the equations of the planes that pass through three points  $(1,1,-1)$ ,  $(6,4,-5)$ ,  $(-4,2,3)$ . 6

- 7 If  $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$   
and  $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$  are any two lines. Find
- D.R. of 1<sup>st</sup> line
  - D.R. of 2<sup>nd</sup> line
  - Angle between the lines
  - If lines are  $\perp$  then  $p = ?$
- 8 If  $\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$   
and  $\vec{r} = (2+\lambda)\hat{i} + (3-\lambda)\hat{j} + 2\hat{k}$  are any two lines then :-
- Find the D.R. of both lines
  - Find Point on both lines through which they are passing
  - Find  $\vec{b}_1 \times \vec{b}_2$  (where  $\vec{b}_1$  &  $\vec{b}_2$  are D.R. of 1<sup>st</sup> and 2<sup>nd</sup> line resp.)
  - Find shortest distance between them
- 9 If  $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$  and  $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$  are any two lines &  $\theta$  is angle between them.
- Write down the formula for  $\cos\theta$
  - Write down the formula for  $\sin\theta$
  - Write the condn for both lines are  $\parallel$
  - Write down the condn for  $\perp$
- 10 If A(1,2,3), B(1,5,3) & C(2,0,-9) are any three pts. Then
- Find D.R. of AB
  - Find D.R. of BC
  - Find D.R. of AC
  - Find angle between AB & BC
  - Find angle between AB & AC
- 11 Find image of the pt. (1,2,1) with respect to the line  $\frac{x-1}{4} = \frac{y+3}{2} = \frac{z-5}{1}$
- 12 Find the image of the point (1,-1,1) with respect to the plane  $2x+3y+z=5$