

# UNIT – XI

## Mathematics

### VECTOR ALGEBRA

#### Group- A

(1 mark each)

- 1 Find direction cosines of the vector  $\vec{a} = 2\hat{i} + 2\hat{j} - \hat{k}$  1
- 2 Find the scalar components of the vector  $\vec{a} = 7\hat{i} + 2\hat{j} - 3\hat{k}$  1
- 3 Find the unit vector along  $\vec{a} = 3\hat{i} + 4\hat{j} - 5\hat{k}$  1
- 4 Find the vector joining the points P(2,3,0) and Q (-1,-2,-4) directed from P to Q. 1
- 5 Find the position vector of the mid point of the line joining the points A (2,3,4) and B (4,1,-2) 1
- 6 Write the formula for finding the projection of  $\vec{a}$  on  $\vec{b}$ . 1
- 7 If  $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$  then find  $|\vec{a}|$ . 1
- 8 If  $\vec{a}$  is  $\perp$  to  $\vec{b}$  then  $\vec{a} \cdot \vec{b} =$  \_\_\_\_\_ 1
- 9 If  $\vec{a}$  is  $\perp$  to  $\vec{b}$  then  $\vec{a} \times \vec{b} \cong$  \_\_\_\_\_ 1
- 10 If  $\theta$  is the angle between  $\vec{a}$  and  $\vec{b}$  then write the expression for  $\vec{a} \cdot \vec{b}$  1
- 11 If  $\theta$  is the angle between  $\vec{a}$  and  $\vec{b}$  then write the expression for  $\vec{a} \times \vec{b}$  1
- 12 Evaluate  $\hat{i} \cdot (\hat{j} \times \hat{k}) + \hat{j} \cdot (\hat{i} \times \hat{k}) + \hat{k} \cdot (\hat{i} \times \hat{j})$  1
- 13 Find  $\vec{AB}$  where A and B are (3,0,5) and (7,1,3) respectively. 1
- 14 If  $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$  then find the angle between  $\vec{a}$  and  $\vec{b}$ . 1
- 15 If  $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ ,  $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$  and  $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$  find  $2\vec{a} - \vec{b} + 3\vec{c}$ . 1
- 16 Find the value of x for which  $x(\hat{i} + \hat{j} + \hat{k})$  is a unit vector. 1
- 17 Write the formula for finding vector area of triangle having  $\vec{a}$  and  $\vec{b}$  as its two adjacent sides. 1
- 18 Write the formula for finding scalar area of a parallelogram having  $\vec{d}_1$  and  $\vec{d}_2$  as its two diagonals. 1

#### Group- B

(4 mark each)

- 1 Find a unit vector in the direction of the vector  $\vec{a} + \vec{b}$  where  $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$  and  $\vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$  1+2+1=4
- 2 Find a vector in the direction of the vector  $3\hat{i} - \hat{j} + 4\hat{k}$ , which has magnitude 6 units. 1+2+1=4
- 3 Let the points A,B,C with position vectors  $\vec{a} = 3\hat{i} - 4\hat{j} - 4\hat{k}$ ,  $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$  and  $\vec{c} = \hat{i} - 3\hat{j} - 5\hat{k}$  respectively form the vertices of a triangle then find- 1+2+1=4
  - (i)  $\vec{AB}$
  - (ii)  $\vec{BC}$
  - (iii)  $\vec{AC}$
  - (iv) Show that A,B,C the vertices of a right angled triangle.
- 4 Find the direction ratio and direction cosines of the vector  $\vec{r} = 2\hat{i} + 3\hat{j} + \hat{k}$  1+2+1=4

- 5 Find the position vector of a point R which divides the line joining A (-2,1,3) and B (3,5,-2) in the ratio 2:1  
 (i) Internally 2+2=4  
 (ii) Externally
- 6 If  $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$ ,  $\vec{b} = 2\hat{i} + 2\hat{j} + \hat{k}$  and  $\vec{c} = 3\hat{i} + \hat{j}$  are such that  $\vec{a} + \lambda \vec{b}$  is perpendicular to  $\vec{c}$ , then find the value of  $\lambda$ . 1+2+1=4
- 7 Find the angle between two vectors  $\vec{a}$  and  $\vec{b}$  with magnitudes  $\sqrt{3}$  and 2, respectively having  $\vec{a} \cdot \vec{b} = \sqrt{6}$  1+2+1=4
- 8 Using vectors, Find the area of  $\Delta ABC$ , whose vertices are A (1,2,3), B (2,5,-1) and C (-1,1,2) 1+1+1+1=4
- 9 Find the area of the parallelogram whose diagonals are represented by the vector  $\vec{d}_1 = 3\hat{i} + \hat{j} - 2\hat{k}$  and  $\vec{d}_2 = \hat{i} - 3\hat{j} + 4\hat{k}$   $\vec{d}_1$  &  $\vec{d}_2$
- 10 Prove that  $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b}) = \vec{0}$  2+2=4
- 11 Let  $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$  and  $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ , Find  $\vec{a} \cdot \vec{b}$  and the projection of  $\vec{a}$  on  $\vec{b}$  2+2=4
- 12 If A (2,3,4), B (5,4,1), C (3,6,2) and D (1,2,0) be four points, show that  $\vec{AB}$  is perpendicular to  $\vec{CD}$ . 1+1+1+1=4
- 13 Find the angle between the vectors  $(\vec{a} + \vec{b})$  and  $(\vec{a} - \vec{b})$ , if  $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$  and  $\vec{b} = 3\hat{i} + \hat{j} + 2\hat{k}$  1+1+1+1=4
- 14 If  $\vec{a}, \vec{b}, \vec{c}$  are unit vectors such that  $\vec{a} + \vec{b} + \vec{c} = \vec{0}$  then find the value of  $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$  1+1+1+1=4
- 15 If  $\vec{a}$  makes equal angles with  $\hat{i}, \hat{j}, \hat{k}$  and has magnitude 3 then prove that the angle between  $\vec{a}$  & each of  $\hat{i}, \hat{j}, \hat{k}$  is  $\cos^{-1}(1/\sqrt{3})$  1+1+1+1=4

### Group- C

(6 mark each)

- 1 Let  $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$ ,  $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ , and  $\vec{c} = 2\hat{i} - \hat{j} + \hat{k}$ . Find a vector  $\vec{d}$  which is perpendicular to both  $\vec{a}$  and  $\vec{b}$  such that  $\vec{c} \cdot \vec{d} = 15$  6
- 2 The scalar product of the vector  $\hat{i} + \hat{j} + \hat{k}$  with a unit vector along the sum of vector  $2\hat{i} + 4\hat{j} - 5\hat{k}$  and  $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$  is equal to one. Find the value of  $\lambda$  6
- 3 Given  $\vec{a} = 3\hat{i} - \hat{j} + \hat{k}$  and  $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$  then express  $\vec{b}$  in the form  $\vec{b} = \beta_1 \vec{a} + \beta_2 \vec{c}$  where  $\beta_1$  is parallel to  $\vec{a}$  and  $\beta_2$  is perpendicular to  $\vec{a}$ . 6
- 4 The two adjacent sides of a parallelogram are  $2\hat{i} - 4\hat{j} + 5\hat{k}$  and  $\hat{i} - 2\hat{j} - 3\hat{k}$ . Find the unit vector parallel to its diagonal. Also find its area. 6
- 5 The dot product of a vector with vectors  $\hat{i} + \hat{j} - 3\hat{k}$ ,  $\hat{i} + 3\hat{j} - 2\hat{k}$  and  $2\hat{i} + \hat{j} + 4\hat{k}$  are 0, 5 and 8 respectively. Find the Vector. 6
- 6 If  $\vec{a} = \hat{i} + \hat{j} + \hat{k}$  and  $\vec{b} = \hat{j} - \hat{k}$  find a vector  $\vec{c}$  such that  $\vec{a} \times \vec{c} = \vec{b}$  and  $\vec{a} \cdot \vec{c} = 3$  1+2+2+1=6
- 7 Find the sine of the angle between the vector  $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$  and  $\vec{b} = \hat{i} + 3\hat{j} + 2\hat{k}$ . Also find projection of  $\vec{a}$  on  $\vec{b}$ . 2+2+2=6

- 8 If a unit vector  $\vec{a}$  makes angles  $\pi/3$  with  $\hat{i}$ ,  $\pi/4$  with  $\hat{j}$  and an acute angle  $\theta$  with  $\hat{k}$  then find  $\theta$  and hence, the components of  $\vec{a}$  1+1+1+  
1+1+1=6
- 9 The vertices of a  $\Delta ABC$  are  $A = (1,-2,-8)$ ,  $B = (5,0,-2)$  and  $C = (11,3,7)$ .  
Find 6
- (a)  $\vec{AB}$  and  $\vec{AC}$  (1.5)
- (b) a unit vector along  $\vec{AB}$  (1.5)
- (c)  $\vec{AB} \times \vec{AC}$  (1.5)
- (d) Scalar area of  $\Delta ABC$  (1.5)
- 10 If  $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$  and  $\vec{b} = 3\hat{i} + 2\hat{j} - \hat{k}$  then find 6
- (a)  $\vec{a} + 3\vec{b}$  (1)
- (b)  $2\vec{a} - \vec{b}$  (1)
- (c)  $(\vec{a} + 3\vec{b}) \cdot (2\vec{a} - \vec{b})$  (1)
- (d) Projection of  $\vec{a}$  on  $\vec{b}$  (1)
- (e) angle between  $\vec{a}$  and  $\vec{b}$  (1)
- (f) projection of  $\vec{b}$  on  $\vec{a}$  (1)