

UNIT-X

Mathematics

(DIFFERENTIAL EQUATIONS)

Very Short answer questions : each carrying one mark.

Objective Questions

1. The degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin(y) + 1 = 0 \text{ is .}$$

2. The degree of the differential equation

$$y'''' + 2y'' + y' = 0$$

3. The order of the differential equation

$$2x^2 \frac{d^2y}{dx^2} - 3 \frac{dy}{dx} + y = 0 \text{ is}$$

4. The order of the differential equation

$$\frac{d^2y}{dx^2} = \cos 3x + \sin 3x$$

5. The number of arbitrary constants in the general solution of differential equation of fourth order is :

6. The number of arbitrary constants in the particular solution of a differential equation of third order are :

7. Is the function $y = \sin x + 1$ is a solution of the differential equation $y'' + y = 0$

8. Is the function $y = e^{-3x}$ is a solution of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$

9. Is the differential equation $\frac{d^2y}{dx^2} + y = 0$ satisfied

10. The general solution of the differential equation $\frac{dy}{dx} = e^x$ is:
11. Find the general solution of $\frac{dy}{dx} = \sin x$.
12. A homogeneous differential equation of the form $\frac{dx}{dy} = h\left(\frac{x}{y}\right)$ can be solved by making the substitution
 (a) $y = vx$ (b) $v = yx$ (c) $x = vy$ (d) $x = v$
13. Write down the standard form of homogeneous equation of degree n .
14. The Integrating factor of the differential equation
 $x \frac{dx}{dy} - y = 2x^2$ is
15. The Integrating factor of the differential equation
 $(1-y^2) \frac{dx}{dy} + yx = ay$ ($-1 < y < 1$) is
16. Write down the standard form of first order linear differential equation.
17. Write the Integrating factor (I.F) of linear equation $\frac{dy}{dx} + Py = Q$ is :
18. Write the Integrating factor of linear equation
 $\frac{dx}{dy} + Px = Q$
19. Order and Degree (if defined) of differential equation are always
 (a) Positive Integer (b) Negative Integer (c) Can't say (d) None of these
20. Find the order of differential equation
 $(y''')^2 + (y'')^3 + (y')^4 + y^5 = 0$

Short answered questions : each carrying 04 (four) marks.

1. Verify that $y = A \cos x - B \sin x$ is a solution of differential equation

$$\frac{d^2y}{dx^2} + y = 0$$

2. Find the differential equation of the family of all hyperbola having foci on the X axis and the centre at the origin.

3. Solve $\frac{dx}{dy} = y \tan x$, when $x = 0$

4. Solve $x \log x \frac{dy}{dx} + y = 2 \log x$

5. Find the particular solution of the following differential equation.

$$\frac{dy}{dx} + y \cot x = 4x \operatorname{cosec} x \quad (x \neq 0) \text{ given that } y = 0 \text{ at } x = \frac{\pi}{2}$$

6. Solve the differential equation

$$\frac{dy}{dx} = \log(x + 1)$$

7. Find the equation of the curve whose differential equation is $(1 + y^2)dx - xydy = 0$ and which passes through $(1, 0)$

8. Solve $x^2 \frac{dy}{dx} = 2xy + y^2$

9. Solve $(1 + x^2) \frac{dy}{dx} + y = \tan^{-1}x$.

10. Obtain the equation of the curve whose slope at any point is equal to $y + 2x$ and which passes through origin.

11. A population grows at the rate of 5% per year. How long does it take for the population to double ?

12. Solve the differential equation

$$\frac{dy}{dx} = \frac{1 - \cos x}{1 + \cos x}$$

13. Solve the differential equation

$$y \frac{dx}{dy} = x + 2y^3$$

14. Verify that $y = e^{m\sin^{-1}x}$ is a solution of the differential equation

$$(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - m^2y = 0$$

15. Find the differential equation of the family of all circles in the second quadrant and touching the coordinate axes.

Long answered questions : each carrying 06 marks.

1. Form the differential equation representing the family of ellipse having foci on x-axis and centre at the origin.

2. Form the differential equation of the family of circles touching the x-axis at the origin.

3. Show that the differential equation $x \cos\left(\frac{y}{x}\right) \frac{dy}{dx} = y \cos\left(\frac{y}{x}\right) + x$ is homogeneous and solve it.

4. Solve $\left(1 + e^{\frac{x}{y}}\right) dx + e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right) dy = 0$

5. Find the general solution of the differential equation

$$x \cdot \frac{dy}{dx} + 2y = x^2 \quad (x \neq 0)$$

6. Find the general solution of the differential equation

$$y dx - (x + 2y^2) dy = 0$$

7. Find the particular solution of the differential equation

$$\frac{dy}{dx} = -4xy^2 \text{ given that } y = 1, \text{ when } x = 0$$

8. (a) Write down the standard form of linear differential equation.

- (b) Compare the given equation $\frac{dy}{dx} + y \cot x = 2x + x^2 \cot x$ with linear differential equation and find P and Q .

- (c) Find Integrating factor (I.F.) of above linear differential equation.

(d) Solve above equation.

9. (a) Write down the standard form of homogeneous differential equation.

(b) Express $(x - y) \frac{dy}{dx} = x + 2y$ in the form of homogeneous differential equation.

(c) Write standard substitution.

(d) Solve above equation.

10. Solve $(1 + x^2) \frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$, $y = 0$ when $x = 0$